arXiv:0905.4425v1 [gr-qc] 27 May 2009

# Horizons in matter: black hole hair vs. Null Big Bang

K. A. Bronnikov
*Center for Gravitation and Fundamental Metrology,*
*VNIIMS, 46 Ozyornaya Street, Moscow 119361, Russia;*
*Institute of Gravitation and Cosmology, PFUR,*
*6 Miklukho-Maklaya Street, Moscow 117198, Russia*[*]

Oleg B. Zaslavskii
*Astronomical Institute of Kharkov V.N. Karazin National University,*
*35 Sumskaya St., Kharkov, 61022, Ukraine*[†]

(Dated: May 27, 2009)

*(Essay written for the Gravity Research Foundation 2009 Awards for Essays on Gravitation)*

It is shown that only particular kinds of matter (in terms of the "radial" pressure to density ratio $w$) can coexist with Killing horizons in black-hole or cosmological space-times. Thus, for arbitrary (not necessarily spherically symmetric) static black holes, admissible are vacuum matter ($w = -1$, i.e., the cosmological constant or some its generalization) and matter with certain values of $w$ between 0 and $-1$, in particular, a gas of disordered cosmic strings ($w = -1/3$). If the cosmological evolution starts from a horizon (the so-called Null Big Bang scenarios), this horizon can co-exist with vacuum matter and certain kinds of phantom matter with $w \geq -3$. It is concluded that normal matter in such scenarios is entirely created from vacuum.

PACS numbers: 04.70.Bw, 04.20.Gz, 04.40 Nr

There are well-known issues in gravitation theory which, upon careful treatment, exhibit unexpected gaps. In black hole physics, the famous no-hair theorems apply to certain kinds of fields (electromagnetic or other gauge fields, dilaton scalars etc.) [1]. Meanwhile, much simpler but physically and astrophysically more relevant environments, namely, macroscopic matter with certain pressure and density, drop out from consideration. Nonetheless, it turns out that if we

**(i)** specify the equation of state near the horizon,

**(ii)** use the Einstein equations and the conservation law and

**(iii)** employ the horizon regularity condition,

we obtain that black holes do admit "matter hair" but only its very specific kinds.

There is a cosmological counterpart of this issue. Looking what can happen beyond a black-hole horizon but very close to it, we can obtain similar conditions that restrict possible types of cosmological scenarios starting from a horizon, the so-called Null Big Bang (NBB) scenarios. There is some complementarity between two seemingly quite different issues, black hole hair and NBB

[*]Electronic address: kb20@yandex.ru
[†]Electronic address: ozaslav@kharkov.ua

scenarios. It is formally expressed in terms of the pressure to density ratio $w$: the above two situations are connected by the relationship $w \leftrightarrow 1/w$.

Let us begin with **black holes** and consider the general static, spherically symmetric metric in the form

$$ds^2 = A(u)dt^2 - \frac{du^2}{A^2(u)} - r^2(u)(d\theta^2 + \sin^2\theta\, d\phi^2) \tag{1}$$

Horizons of order $n$ correspond to regular zeros of $A(u)$: $A \sim (u - u_h)^n$, $n \in \mathbb{N}$.

The stress-energy tensor (SET) of matter can be written as

$$T_\mu^\nu = \mathrm{diag}\ (\rho, -p_r, -p_\perp, -p_\perp). \tag{2}$$

Then the combination $\binom{0}{0} - \binom{1}{1}$ of the Einstain equations shows that on the horizon

$$p_r(u_h) + \rho(u_h) = 0, \tag{3}$$

so the null energy condition (NEC) $T_\mu^\nu \xi^\mu \xi_\nu \geq 0$, $\xi^\mu \xi_\mu = 0$ is satisfied on the verge. One can note that matter characterized by the condition (3) in the whole space is a natural generalization of the cosmological constant in that its SET is invariant under radial boosts [2], and, following [3], we call it vacuum matter. Without losing much generality, we assume that for our matter near the horizon $p_r = w\rho$, $w = \mathrm{const}$, $\rho \geq 0$. If the NEC holds ($w \geq -1$), we call the matter normal, otherwise it is said to be phantom. We also admit a noninteracting admixture of vacuum matter.

An analysis according to the above items (i)–(iii) leads to the following results [4].

**Theorem 1**. A static, spherically symmetric black hole can be in equilibrium with a static matter distribution with the SET (2) only if near the event horizon either (a) $w \to -1$ (as in vacuum matter) or (b) $w \to -1/(1+2k)$, $k \in \mathbb{N}$. In case (a), the horizon can be of any order $n$ and $\rho(u_h) \neq 0$. In case (b), the horizon is simple ($n = 1$), and $\rho \sim u - u_h$.

The generic case of such hairy black holes is $k = 1$ implying $w = -1/3$, which, for $p_\perp = p_r$, corresponds to a fluid of disordered cosmic strings. Since such strings may be arbitrarily curved or closed, one can partly express the meaning of the theorem by the words "black holes can have curly hair".

In the presence of vacuum matter, the following theorem holds:

**Theorem 2**. A static, spherically symmetric black hole can be in equilibrium with a noninteracting mixture of static nonvacuum matter with the SET (2) and vacuum matter with the SET (2) only if near the event horizon

$$w \to -n/(n + 2k) \tag{4}$$

where $n$ is the order of the horizon, $n \leq k$, and $\rho \sim (u - u_h)^k$.

Any amount of other kinds of matter, normal or phantom, added to such a configuration, should break its static character by simply falling onto the horizon or maybe even destroying the black hole. In other words, black holes may be hairy, or dirty, but, in the near-horizon region, normal ($p_r \geq 0$) or phantom ($p_r < -\rho$) hair are completely excluded. In an equilibrium configuration, all "dirt" is washed away from the near-horizon region, except vacuumlike or modestly exotic, probably "curly" hair. In particular, a static black hole cannot live inside a star of normal matter with $p_r \geq 0$ unless there is an accretion region around the horizon or a layer of string and/or vacuum matter.

Our approach is also relevant to semiclassical black holes in equilibrium with their Hawking radiation (the Hartle-Hawking state), whose SET essentially differs from that of a perfect fluid. Since the density of quantum fields is, in general, nonzero at the horizon (see Sec. 11 of the textbook [1] for details), by the regularity condition (3), such quantum radiation should behave near the horizon like vacuum matter. A mixture of Hawking radiation and some kinds of classical matter with $-1 < w < 0$ is also admissible.

**Null Big Bang.** Spherically symmetric cosmological models are characterized by the general Kantowski-Sachs (KS) metric

$$ds^2 = b^2(t)dt^2 - a^2(t)dx^2 - r^2(t)(d\theta^2 + \sin^2\theta d\phi^2). \tag{5}$$

Using the "quasiglobal" gauge ($b(t) = 1/a(t)$ and assuming a horizon at some $t = t_h$, so that $r(t_h)$ is finite and

$$a^2(t) \sim (t - t_h)^n, \quad n \in \mathbb{N}, \tag{6}$$

we can study the near-horizon behavior of the system in the above manner. Then the following general results are obtained [5]:

1. With any normal matter, regular cosmological evolution can only begin with a Killing horizon.
2. Noninteracting normal matter cannot exist in a KS cosmology near a horizon.
3. Normal matter can only appear after a NBB due to interaction with a sort of vacuum.

More specifically, the parameter $w$ should obey the condition $w = -1 - 2k/n$ where $k \geq n > 0$, $k, n \in \mathbb{N}$. It can be obtained from (4) by replacing $w \to 1/w$.

This generalizes the conclusions made in [3] for KS cosmologies with dustlike matter. As for phantom matter, in its presence a nonsingular cosmology is posiible without a NBB. But if there is a horizon, it is compatible with phantom matter with $w \leq -3$ only.

In NBB scenarios, the Universe is highly anisotropic right after crossing the horizon, since one of its scale factors, $a(t)$, evolves from zero whereas the other, $r(t)$, is finite. So, at this stage, intense particle creation from vacuum should occur, leading to rapid isotropization [6, 7]. Estimates on a phenomenological level [3] show that that at least some NBB scenarios automatically lead to sufficient isotropy at later stages of evolution. This confirms the potential viability of this kind of models. Moreover, the existence of a pre-horizon static stage, which is observable from the cosmological stage, makes all cosmological events causally connected thus removing the inherent difficulties of usual Big Bang scenarios, conventionally solved by inflation.

**Abandoning spherical symmetry.** It is generally assumed that, in the course of gravitational collapse to a black hole, nonspherical perturbatons die out. However, in astrophysical conditions (e.g., in double or multiple stellar systems) even static black holes, being surrounded by nonsymmetric matter distributions, deflect from spherical symmetry. In such more general black holes, generic properties of Killing horizons should play an important role in a (not yet quite well understood) relationship between the symmetry of horizons and black hole entropy (see, e.g., [8] and references therein).

It turns out, however [9], that almost all restrictions on the values of the parameter $w$ compatible with horizons, described above, hold in the general static case. Such results are obtained with the

above items (i)–(iii), now applied to a general static metric written with the aid of the Gaussian coordinate $l$, orthogonal to a family of 2-surfaces, one of which ($l = l_h$) corresponds to the horizon:

$$ds^2 = -N^2 dt^2 + dl^2 + \gamma_{ab} dx^a dx^b, \qquad a, b = 2, 3. \tag{7}$$

where $N = N(l, x^2, x^3)$ and $\gamma_{ab} = \gamma_{ab}(l, x^2, x^3)$. This metric was used in [8] in discussing regularity conditions on the horizon, and the relation (3) was obtained, where $p_r$ now denotes pressure in the $l$ direction).

As in spherically symmetric configurations, the possible existence or non-existence of a black hole inside matter and the horizon type depend on the value of $w$ and on the presence or absence of vacuum matter with the density $\rho_{\rm vac}$, and analogs of Theorems 1 and 2 are valid. Nevertheless, some new features do appear [9]. Thus, first, in the absence of vacuum matter, degenerate horizons ($n \geq 2$) are now possible if the corresponding 2-metric $\gamma_{ab}$ is flat, which is true, e.g., for cylindrical black holes. Second, if $\rho_{\rm vac} \neq 0$, the sign of $\rho_{\rm vac}(l_h)$ coincides with that of two-dimensional curvature of the horizon. In particular, $\rho_{\rm vac} < 0$, forbidden in the spherical case, is compatible with a horizon with hyperbolic geometry. Thus matter with certain negative values of $w$ can contain horizons with non-spherical topologies (cf. [10]).

Horizons in nonsymmetric NBB cosmologies can be considered in a similar way, with almost the same results as described here, to be presented in detail elsewhere. It would be of interest to obtain explicit examples of non-KS nonsingular NBB scenarios. In the KS case, such examples are already known, one with phenomenological vacuum matter [3] and another with a phantom scalar field [11].

To conclude, in our reasoning, which has been entirely local and relied on near-horizon expansions, we did not assume any particular equation of state for matter and even did not restrict the behaviour of the transverse pressure except for its regularity requirement. In this sense, our conclusions are model-independent.

---


[1] V. P. Frolov and I. D. Novikov, *Black Hole Physics: Basic Concepts and New Developments* (Kluwer Academic, Boston, 1998).

[2] I.G. Dymnikova, Gen. Rel. Grav. **24**, 235 (1992).

[3] K.A. Bronnikov and I.G. Dymnikova, Class. Quantum Grav. **24**, 5803 (2007).

[4] K.A. Bronnikov and O.B. Zaslavskii, Phys. Rev. D **78**, 021501 (R) (2008).

[5] K.A. Bronnikov and O.B. Zaslavskii, Class. Quantum Grav. **25**, 105015 (2008).

[6] A.A. Grib, S.G. Mamaev and V.M. Mostepanenko, *Quantum Effects in Strong External Fields* (Atomizdat, Moscow, 1980, in Russian).

[7] N. Birrell and P. Davies, *Quantum Fields in Curved Space* (Cambridge Univ. Press, 1982).

[8] A.J.M. Medved, D. Martin and M. Visser, Class. Quantum Grav. **21**, 3111 (2004).

[9] K.A. Bronnikov and O.B. Zaslavskii, General static black holes in matter, ArXiv: 0904.4904.

[10] D.R. Brill, J. Louko and Peldan, Phys. Rev. D **56**, 3600 (1997);
L. Vanzo, Phys. Rev. D **56**, 6475 (1997);
J.P.S. Lemos, Phys. Rev. D **57**, 4600 (1998).

[11] K.A. Bronnikov and J.C. Fabris, Phys. Rev. Lett. **96**, 251101 (2006).